

P 35

a) A: $\lambda_1 = 3$ $v_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ $\lambda_2 = -4$ $v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

B: $\lambda_{1,2} = 2$ $v_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

C: $\lambda_1 = +i$ $v_1 = \begin{pmatrix} 3+i \\ -5 \end{pmatrix}$ $\lambda_2 = -i$ $v_2 = \begin{pmatrix} 3-i \\ -5 \end{pmatrix}$

P 36

a) $A v_1 = \begin{pmatrix} 8 \\ -8 \\ 8 \\ 0 \end{pmatrix} \Rightarrow v_1$ kein EV

$v_2 = \vec{0}$ per Def. nie EV

$A v_3 = \begin{pmatrix} 30 \\ 30 \\ 0 \\ 20 \end{pmatrix} = 10 v_3 \Rightarrow v_3$ EV zum EW 10

b) $\det(A - \lambda E) = \begin{vmatrix} 12-\lambda & 4 & 0 & -9 \\ 2 & 10-\lambda & 0 & -3 \\ 2 & 2 & 1-\lambda & -6 \\ 2 & 2 & 0 & 4-\lambda \end{vmatrix}$
 $= (1-\lambda)(-\lambda^3 + 26\lambda^2 - 224\lambda + 640)$
 $= (1-\lambda)(10-\lambda)(8-\lambda)^2$

$\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \begin{pmatrix} 3 \\ 3 \\ 0 \\ 2 \end{pmatrix}; \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$

$\lambda_1 = 1 \quad \lambda_2 = 10 \quad \lambda_3 = 8$

c) $e_{\lambda_1} = d_{\lambda_1} = 1; e_{\lambda_2} = d_{\lambda_2} = 1; e_{\lambda_3} = 2, d_{\lambda_3} = 1$

P 37

v EV zum EW λ , $\varphi(v) = \lambda v$

$v = (\varphi \circ \varphi)(v) = \varphi(\varphi(v)) = \varphi(\lambda v) = \lambda^2 v$

$\Rightarrow \lambda^2 = 1, \lambda = \pm 1$

Zusatz: $w = \varphi(v)$

$\varphi(w) = \varphi(\varphi(v)) = v$

$\varphi\left(\frac{v}{2} + \frac{w}{2}\right) = \frac{1}{2} \varphi(v) + \frac{1}{2} \varphi(w) = \frac{1}{2} v + \frac{1}{2} w$

$\varphi\left(\frac{v}{2} - \frac{w}{2}\right) = \frac{1}{2} \varphi(v) - \frac{1}{2} \varphi(w) = \frac{1}{2} w - \frac{1}{2} v = -\left(\frac{1}{2} v - \frac{1}{2} w\right)$

Falls $v = -w$ oder $v = w \Rightarrow v$ EV zum EW -1 bzw 1

Ganz sind $\frac{1}{2}(v+w)$ und $\frac{1}{2}(v-w)$ EV und v deren
Summe.