P32

a) $\alpha(g) = \begin{pmatrix} 1 & 0 & 1 \\ 1 & -1 & -1 \\ 0 & 1 & 2 \end{pmatrix} \left(\lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} \right) + \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$

$= \lambda \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \\ -1 \end{pmatrix}$

$\beta(g) = \begin{pmatrix} 3 & 0 & -4 \\ 1 & 2 & -1 \\ 2 & 1 & 0 \end{pmatrix} \left(\lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} \right) + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$= \lambda \begin{pmatrix} 7 \\ 6 \\ 4 \end{pmatrix} + \begin{pmatrix} 9 \\ -1 \\ 4 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \lambda \begin{pmatrix} 7 \\ 6 \\ 4 \end{pmatrix} + \begin{pmatrix} 9 \\ 0 \\ 4 \end{pmatrix}$

b) $\alpha \circ \beta : v \mapsto A(B \cdot v + r) + t = \begin{pmatrix} 1 & 0 & 1 \\ 1 & -1 & -1 \\ 0 & 1 & 2 \end{pmatrix} \left(\begin{pmatrix} 3 & 0 & -4 \\ 1 & 2 & -1 \\ 2 & 1 & 0 \end{pmatrix} v + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right) + \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 5 & 1 & -4 \\ 0 & -3 & -3 \\ 5 & 4 & -1 \end{pmatrix} v + \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \underbrace{\begin{pmatrix} 5 & 1 & -4 \\ 0 & -3 & -3 \\ 5 & 4 & -1 \end{pmatrix}}_{\text{lin. Anteil}} v + \underbrace{\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}}_{\text{Translationsanteil}}$

$\beta \circ \alpha : w \mapsto B(Aw + t) + r = \begin{pmatrix} 3 & -4 & -5 \\ 3 & -3 & -3 \\ 3 & -1 & 1 \end{pmatrix} w + \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$

P33

$\mathbb{F} = (P_0; P_1 - P_0; P_2 - P_0; P_3 - P_0) = \left(\begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}; \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix}; \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}; \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right)$

${}_{\mathbb{E}}\mathcal{K}_{\mathbb{F}} : v \mapsto w = \underbrace{\begin{pmatrix} 1 & -2 & 0 \\ 3 & 1 & 1 \\ 3 & 2 & 1 \end{pmatrix}}_{A} v + \underbrace{\begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}}_{b}$

${}_{\mathbb{F}}\mathcal{K}_{\mathbb{E}} : w \mapsto v = A^{-1}(w - b) = \begin{pmatrix} 1 & -2 & 2 \\ 0 & -1 & 1 \\ -3 & 8 & -7 \end{pmatrix} w + \begin{pmatrix} -11 \\ -3 \\ 38 \end{pmatrix}$

${}_{\mathbb{E}}P_4 = {}_{\mathbb{E}}\mathcal{K}_{\mathbb{F}}({}_{\mathbb{F}}P_4) = \begin{pmatrix} 1 & -2 & 0 \\ 3 & 1 & 1 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 6 \end{pmatrix}$

P34

a) nicht orthogonal, da z.B. $\langle f_1, f_2 \rangle = -8 \neq 0$

b) $_{\mathbb{F}}\vec{O} = \frac{1}{16}\begin{pmatrix} -97 \\ -61 \\ -3 \end{pmatrix}$; $_{\mathbb{F}}e_1 = \frac{1}{16}\begin{pmatrix} -101 \\ -65 \\ 1 \end{pmatrix}$

$_{\mathbb{F}}e_2 = \frac{1}{16}\begin{pmatrix} -87 \\ -59 \\ -5 \end{pmatrix}$; $_{\mathbb{F}}e_3 = \frac{1}{16}\begin{pmatrix} -75 \\ -47 \\ -1 \end{pmatrix}$

c) $_{\mathbb{E}}K_{\mathbb{F}}: v \mapsto w = Fv + p = \begin{pmatrix} 1 & -2 & 3 \\ 2 & -3 & -1 \\ 0 & 1 & 1 \end{pmatrix} v + \begin{pmatrix} -1 \\ 1/2 \\ 4 \end{pmatrix}$, $F = (f_1 | f_2 | f_3)$

d) $_{\mathbb{F}}K_{\mathbb{E}}: w \mapsto v = F^{-1}(w-p) = \frac{1}{8}\begin{pmatrix} -2 & 5 & 11 \\ -2 & 1 & 7 \\ 2 & -1 & 1 \end{pmatrix} w + \frac{1}{16}\begin{pmatrix} -97 \\ 61 \\ -3 \end{pmatrix}$