P23

$$A_c \rightsquigarrow \begin{matrix} c(c-1) & 0 & 0 & c(c-1) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2c \\ 0 & 0 & 2c & 2c-c^2 \end{matrix} \quad \Big]\cdot(-2c) \; +$$

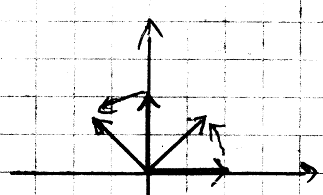
$$\rightsquigarrow \begin{matrix} c(c-1) & 0 & 0 & c(c-1) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2c \\ 0 & 0 & 0 & -5c(c-\frac{2}{5}) \end{matrix}$$

$c \in \mathbb{R} \setminus \{0, 1, \frac{2}{5}\} \quad Rg(A_c) = 4$

$c = 0 \quad Rg(A_0) = 2 \quad \begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$

$c = 1 \quad Rg(A_1) = 3 \quad \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$

P24



a) $\begin{pmatrix}1\\0\end{pmatrix} \to \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

$\begin{pmatrix}0\\1\end{pmatrix} \to \frac{1}{\sqrt{2}}\begin{pmatrix}-1\\1\end{pmatrix} \qquad {}_c\delta_c = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\1 & 1\end{pmatrix}$

b) $b_2 \in E_1 \quad b_3 \in E_1$

$d(b_1, E_1) = 1$

$d(-b_1, E_1) = 1 \qquad {}_B\varphi_B \begin{pmatrix}-1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}$

$d(b_1, -b_1) = 2$

$\begin{pmatrix}1\\0\\0\end{pmatrix} = \frac{1}{9}(3b_1 + 4b_2 + 2b_3)$

$\begin{pmatrix}1\\0\\0\end{pmatrix} \to \frac{1}{9}(-3b_1 + 4b_2 + 2b_3) = \frac{1}{9}\begin{pmatrix}7\\-4\\4\end{pmatrix}$

$\begin{pmatrix}0\\1\\0\end{pmatrix} = \frac{1}{9}(6b_1 - b_2 + 4b_3) \qquad \begin{pmatrix}0\\1\\0\end{pmatrix} \to \frac{1}{9}(-6b_1 - b_2 + 4b_3) = \frac{1}{9}\begin{pmatrix}-4\\1\\8\end{pmatrix}$

$\begin{pmatrix}0\\0\\1\end{pmatrix} = \frac{1}{9}(-6b_1 + b_2 + 5b_3) \qquad \begin{pmatrix}0\\0\\1\end{pmatrix} \to \frac{1}{9}(6b_1 + b_2 + 5b_3) = \frac{1}{9}\begin{pmatrix}4\\8\\1\end{pmatrix}$

$${}_E\varphi_E = \frac{1}{9}\begin{pmatrix}7 & -4 & 4\\-4 & 1 & 8\\4 & 8 & 1\end{pmatrix}$$

P25:

a) $\beta \circ \alpha$: $B \cdot A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 0 & 2 \\ -1 & 3 & 2 \\ 0 & 1 & 1 \end{pmatrix}$ $Rg(BA) = 2$

$\gamma \circ \beta$: $C \cdot B = \begin{pmatrix} 2 & 10 \\ 3 & 2 \\ 6 & 0 \end{pmatrix}$ $Rg(CB) = 2$

$\gamma \circ \beta \circ \alpha$: $CBA = \begin{pmatrix} 2 & 10 & 12 \\ 3 & 2 & 5 \\ 6 & 0 & 6 \end{pmatrix}$ $Rg(CBA) = 2$

b) $Rg(A) = 2$

$Rg(B) = 2$

c) $\alpha: v \mapsto Av$

$\beta: w \mapsto Bw$

$\beta \circ \alpha: v \mapsto B(Av)$

BAv

Wird begrenzt vom Rang der Matrix mit dem geringsten Rang.