Jan Strotbek

Hausübung 12

H38 a)

$$A = \begin{pmatrix} -10 & -2 & 24 \\ 20 & 3 & -40 \\ -4 & -1 & 10 \end{pmatrix} \qquad T = \begin{pmatrix} 1 & 2 & -2 \\ -4 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

$$\begin{array}{ccc|ccc} 1 & 2 & -2 & 1 & 0 & 0 \\ -4 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{array}$$

$$\begin{array}{ccc|ccc} 1 & 2 & -2 & 1 & 0 & 0 \\ 0 & 8 & -7 & 4 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{array}$$

$$\begin{array}{ccc|ccc} 1 & 2 & -2 & 1 & 0 & 0 \\ 0 & 8 & -7 & 4 & 1 & 0 \\ 0 & 0 & -1 & -4 & -1 & 8 \end{array}$$

$$\begin{array}{ccc|ccc} 1 & 2 & 0 & 9 & 2 & -16 \\ 0 & 8 & 0 & 32 & 8 & -56 \\ 0 & 0 & 1 & 4 & 1 & -8 \end{array}$$

$$\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -2 \\ 0 & 1 & 0 & 4 & 1 & -7 \\ 0 & 0 & 1 & 4 & 1 & -8 \end{array} \qquad T^{-1} = \begin{pmatrix} 1 & 0 & -2 \\ 4 & 1 & -7 \\ 4 & 1 & -8 \end{pmatrix}$$

$$B = T^{-1} \cdot A \cdot T = \begin{pmatrix} -2 & 0 & 4 \\ 8 & 2 & -14 \\ -12 & 3 & -24 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & -2 \\ -4 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

b) EW und EV von B

$\lambda_1 = -2 \qquad v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\lambda_2 = 2 \qquad v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\lambda_3 = 3 \qquad v_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

c) EW und EV von A

A hat die gleichen Eigenwerte wie B

$\Rightarrow \quad \lambda_1 = -2 \quad \lambda_2 = 2 \quad \lambda_3 = 3$

$\lambda_1 = -2$

$$\begin{matrix} -8 & -2 & 24 \\ 20 & 5 & -40 \\ -4 & -1 & 12 \end{matrix}$$

$$\begin{matrix} -4 & -1 & 12 \\ 4 & 1 & -8 \\ -4 & -1 & 12 \end{matrix}$$

$$\begin{matrix} -4 & -1 & 12 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{matrix}$$

$$v_1 = \begin{pmatrix} 1 \\ -4 \\ 0 \end{pmatrix}$$

$\lambda_2 = 2$

$$\begin{matrix} -12 & -2 & 24 \\ 20 & 1 & -40 \\ -4 & -1 & 8 \end{matrix}$$

$$\begin{matrix} -6 & -1 & 12 \\ 20 & 1 & -40 \\ -4 & -1 & 8 \end{matrix}$$

$$\begin{matrix} -6 & -1 & 12 \\ 0 & -7 & 0 \\ 0 & -1 & 0 \end{matrix}$$

$$v_2 = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

$\lambda_3 = 3$

$$\begin{matrix} -13 & -2 & 24 \\ 20 & 0 & -40 \\ -4 & -1 & 7 \end{matrix}$$

$$\begin{matrix} -13 & -2 & 24 \\ 0 & -2 & -2 \\ 0 & -5 & -5 \end{matrix}$$

$$v_3 = \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}$$

Die Eigenvektoren von A entsprechen den Spaltenvektoren der Transformationsmatrix T.

H 39

$$A = \frac{1}{64} \cdot \begin{pmatrix} 7 & 3\sqrt{3} & 0 \\ 3\sqrt{3} & 13 & 0 \\ 0 & 0 & -128 \end{pmatrix}$$

$$\chi = (-2-\lambda)\left[\left(\frac{7}{64}-\lambda\right)\left(\frac{13}{64}-\lambda\right) - \frac{27}{64^2}\right]$$

$$= (-2-\lambda)\left(\lambda^2 - \frac{20}{64}\lambda + \frac{1}{64}\right)$$

$$\lambda_1 = \frac{1}{4} \qquad \lambda_2 = \frac{1}{16} \qquad \lambda_3 = -2$$

$$\begin{matrix} -9 & 3\sqrt{3} & 0 \\ 3\sqrt{3} & -3 & 0 \\ 0 & 0 & -2 \end{matrix} \qquad \begin{matrix} 3 & 3\sqrt{3} & 0 \\ 3\sqrt{3} & 9 & 0 \\ 0 & 0 & -2 \end{matrix} \qquad \begin{matrix} 135 & 3\sqrt{3} & 0 \\ 3\sqrt{3} & 141 & 0 \\ 0 & 0 & 0 \end{matrix}$$

$$\begin{matrix} -9 & 3\sqrt{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{matrix} \qquad \begin{matrix} 3 & 3\sqrt{3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{matrix} \qquad \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{matrix}$$

$$v_1 = \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \\ 0 \end{pmatrix} \qquad v_2 = \begin{pmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix} \qquad v_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$T = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad T^T \cdot A \cdot T = \begin{pmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{16} & 0 \\ 0 & 0 & -2 \end{pmatrix} = D$$

Drehwinkel $\varphi = \cos^{-1}\left(\frac{1}{2}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \mathbf{60°}$

$$Q = \{ x \in \mathbb{R}^3 \mid x^T A x = 0 \}$$

Bezüglich D $\quad q(x) = \frac{1}{4}x_1^2 + \frac{1}{16}x_2^2 - 2x_3^2 = 0$

Q ist folglich ein Doppelkegel

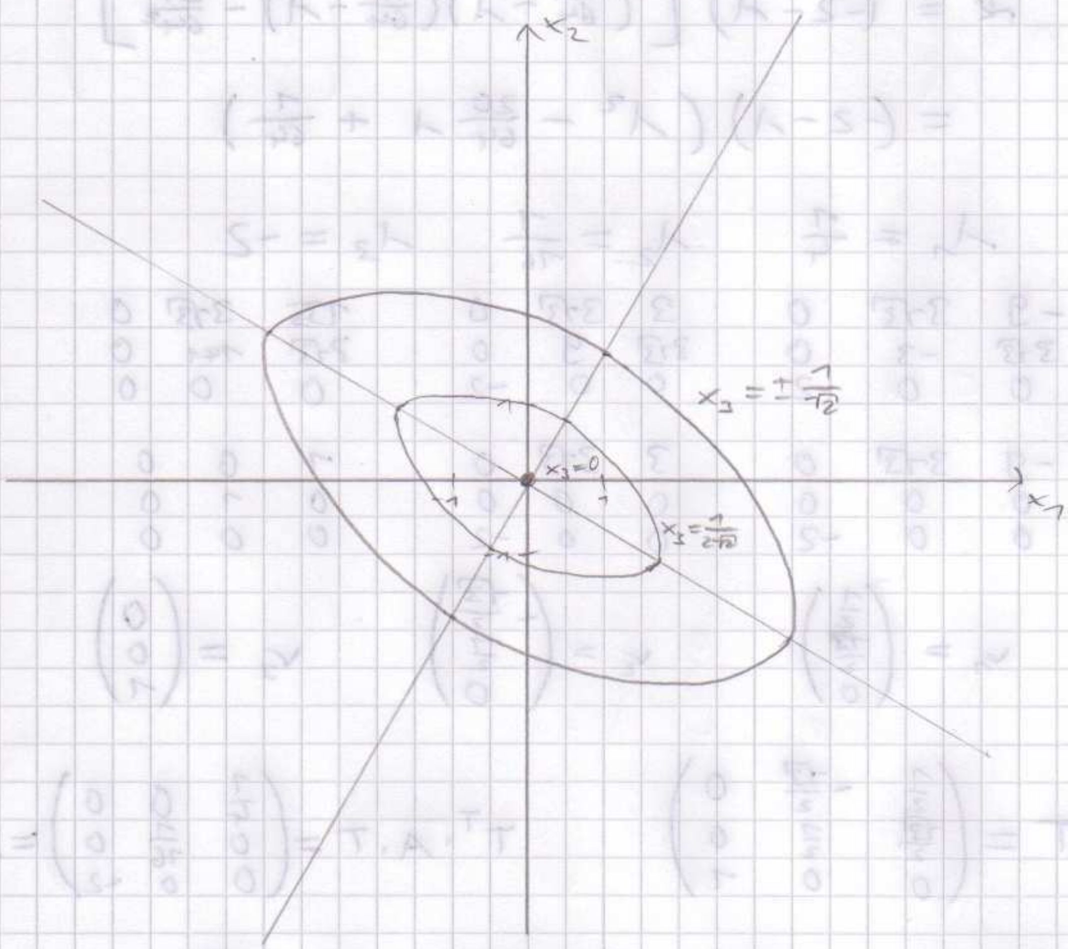
$x_3 = -\frac{1}{\sqrt{2}} \longrightarrow \frac{1}{4}x_1^2 + \frac{1}{16}x_2^2 = 1$ Ellipse

$x_3 = 0 \longrightarrow \frac{1}{4}x_1^2 + \frac{1}{16}x_2^2 = 0$ Punkt(0|0|0)

$x_3 = \frac{1}{2\sqrt{2}} \longrightarrow \frac{1}{4}x_1^2 + \frac{1}{16}x_2^2 = \frac{1}{4}$ Ellipse

$x_3 = \frac{1}{\sqrt{2}} \longrightarrow \frac{1}{4}x_1^2 + \frac{1}{16}x_2^2 = 1$ Ellipse

Die Spitzen des Doppelkegels treffen sich im Punkt (0|0|0). Zur x_1x_2-Ebene parallele Schnittflächen sind elliptisch.



H40

$$Q = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid 3x_1^2 - 2x_2^2 - x_3^2 - 1 = 0\}$$

a) quadratischer Teil $3x_1^2 - 2x_2^2 - x_3^2$

linearer Teil —

Konstanter Teil -1

b)

$$Q = \left\{ x \in \mathbb{R}^3 \;\middle|\; x^T \begin{pmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{pmatrix} x - 1 = 0 \right\}$$

c)

$x_1 = 0 \longrightarrow -2x_2^2 - x_3^2 - 1 = 0$

Kein Punkt

$x_2 = 0 \longrightarrow 3x_1^2 - x_3^2 - 1 = 0$

Hyperbel

$(\sqrt{3}x_1 + x_2)(\sqrt{3}x_1 - x_2) = 1$

$x_3 = 0 \longrightarrow 3x_1^2 - 2x_2^2 - 1 = 0$

Hyperbel

$(\sqrt{3}x_1 + \sqrt{2}x_2)(\sqrt{3}x_1 - \sqrt{2}x_2) = 1$

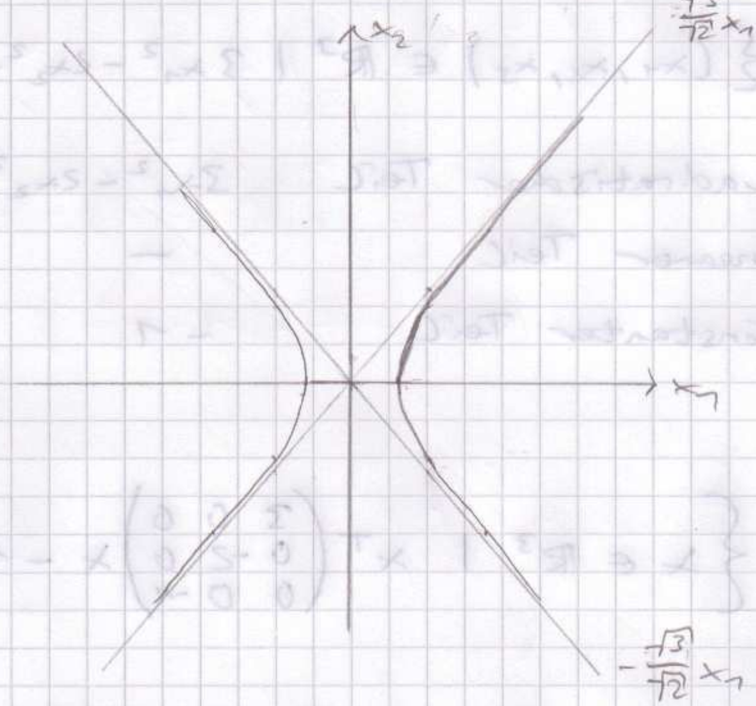
$x_1 = \pm 1 \longrightarrow -2x_2^2 - x_3^2 + 2 = 0$

Ellipse

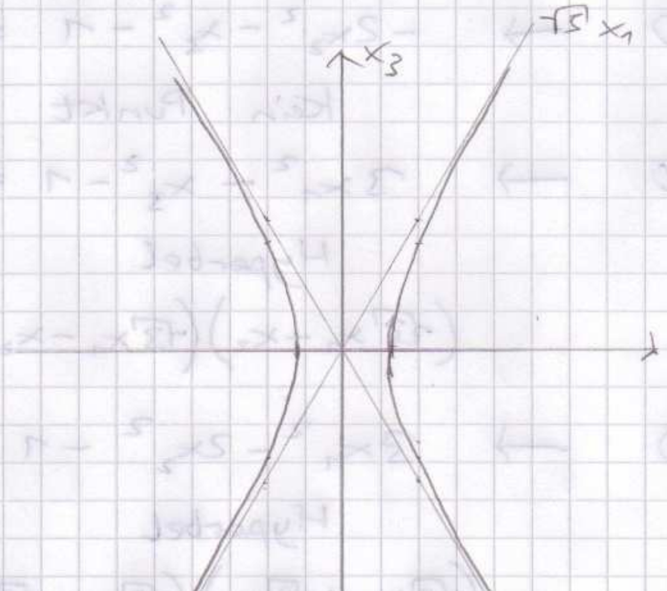
Es handelt sich um eine Mittelpunkts-quadrik mit $M = (0|0|0)$.

BRUNNEN

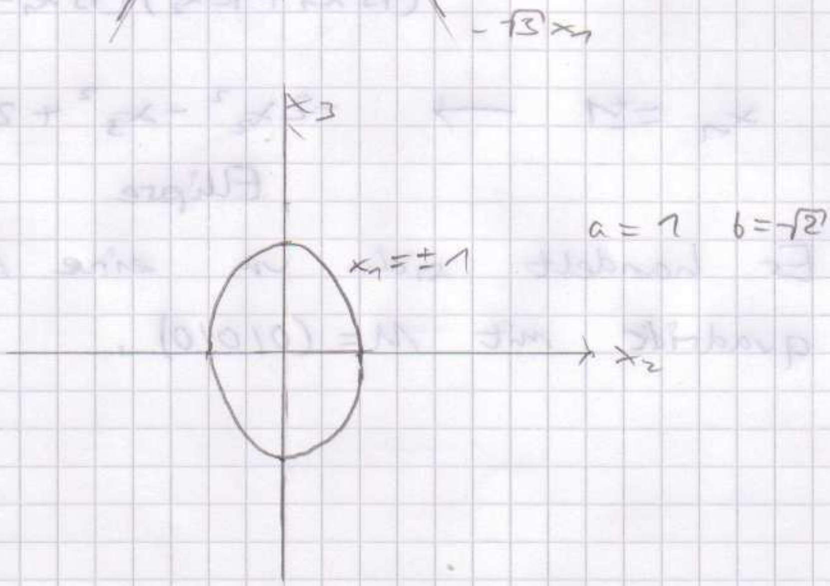
d)



$\frac{\sqrt{3}}{\sqrt{2}}x_1$
x_2
x_1
$-\frac{\sqrt{3}}{\sqrt{2}}x_1$



$\sqrt{3}x_1$
x_3
x_1
$-\sqrt{3}x_1$



x_3
$x_1 = \pm 1$
x_2
$a = 1 \quad b = \sqrt{2}$