Hausübung 11

H35

$$A = \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix} \qquad \lambda_1 = 1 \quad \lambda_2 = -1 \quad \lambda_3 = 2$$

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \quad v_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad v_3 = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$$

a)

A symmetrisch $\Rightarrow$ v_1, v_2, v_3 orthogonal

$$T^T \cdot A \cdot T = B$$

$$T \cdot B \cdot T^T = A$$

T erhält man indem man v_1, v_2, v_3 normiert

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & -1 & -\frac{1}{2} \\ -1 & 1 & -1 \\ -\frac{1}{2} & -1 & \frac{1}{2} \end{pmatrix}$$

b)

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 2 & 1 \\ -1 & 1 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 2 & -2 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & -6 & -2 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{3} \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{3} \end{array}\right)$$

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$$

$$x = \frac{1}{3\lambda_2}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \frac{1}{3\lambda_3}\begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ 0 \\ -\frac{1}{2} \end{pmatrix}$$

H136

a) $A^n \cdot v = \lambda^n \cdot v$

IA $n = 1$

$A^1 \cdot v = \lambda^1 \cdot \cancel{v}$ (Def. für Eigenwert)

IS $n \to n+1$

$$A^{n+1} \cdot v = A \cdot A^n \cdot v = A \cdot \lambda^n \cdot v$$
$$= \lambda \cdot \lambda^n \cdot v$$
$$= \lambda^{n+1} \cdot v$$

q.e.d.

b)

$A = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$ $\lambda_1 = 1$ $\lambda_2 = i$

$A^2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\sqrt[2]{1} = \pm 1$

$\sqrt[2]{-1} = \pm i$

-1 und $-i$ sind keine Eigenwerte von A.

H37

a)

$$A = \begin{pmatrix} \sqrt{2} & -\sqrt{2}i & 0 \\ -\sqrt{2}i & -\sqrt{2} & 0 \\ 0 & 0 & -4 \end{pmatrix}$$

$$\chi = (-4-\lambda)(\lambda^2 - 2 - 2i^2)$$
$$= (-4-\lambda)(\lambda^2)$$

$\lambda_1 = 0 \qquad \lambda_2 = -4$

$\lambda_1 = 0$

$$\begin{matrix} \sqrt{2} & -\sqrt{2}i & 0 \\ -\sqrt{2}i & -\sqrt{2} & 0 \\ 0 & 0 & -4 \end{matrix}$$

$$\begin{matrix} \sqrt{2} & -\sqrt{2}i & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -4 \end{matrix} \quad + i \cdot z_1 \qquad v_1 = \begin{pmatrix} -1 \\ i \\ 0 \end{pmatrix}$$

$\lambda_2 = -4$

$$\begin{matrix} \sqrt{2}+4 & -\sqrt{2}i & 0 \\ -\sqrt{2}i & -\sqrt{2}+4 & 0 \\ 0 & 0 & 0 \end{matrix}$$

Lässt sich durch Gaußumformungen auf

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{matrix}$$ bringen

$$\Rightarrow \qquad v_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$V_{\lambda_1} = s \cdot \begin{pmatrix} -1 \\ i \\ 0 \end{pmatrix} \qquad V_{\lambda_2} = t \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

b) $f_1 = \begin{pmatrix} -1 \\ i \\ 0 \end{pmatrix}$ $f_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$\left(\begin{array}{ccc|c} \sqrt{2} & -\sqrt{2}i & 0 & -1 \\ -\sqrt{2}i & -\sqrt{2} & 0 & i \\ 0 & 0 & -4 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} \sqrt{2} & -\sqrt{2}i & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -4 & 0 \end{array}\right)$$

$$\sqrt{2}\, x_1 - \sqrt{2}\, i\, x_2 = -1$$

$f_2 = \begin{pmatrix} 0 \\ -\frac{i}{\sqrt{2}} \\ 0 \end{pmatrix}$ $A \cdot f_2 = \begin{pmatrix} -1 \\ i \\ 0 \end{pmatrix}$

$$\Rightarrow \; {}_F\varphi_F = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -4 \end{pmatrix}$$