H 29) a) $f: \mathbb{R}^2 \to \mathbb{R}: \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \longmapsto x_1^3 + x_2^3 + 3x_1x_2$

$$\operatorname{grad} f(x_1, x_2) = \left(3x_1^2 + 3x_2 \; ; \; 3x_2^2 + 3x_1\right)$$

$$\begin{aligned} 3x_1^2 + 3x_2 &= 0 \\ 3x_2^2 + 3x_1 &= 0 \end{aligned}$$

$$\begin{aligned} x_1^2 + x_2 &= 0 \\ x_2^2 + x_1 &= 0 \end{aligned} \qquad x_1 = -x_2^2$$

$$\left(-x_2^2\right)^2 + x_2 = 0$$

$$x_2^4 + x_2 = 0$$

$$x_2\left(x_2^3 + 1\right) = 0$$

$x_{21} = 0$ $\quad$ $x_{11} = 0$

$x_{22} = -1$ $\quad$ $\cancel{x_{12} = 1}$

$x_{12} = -1$

$$\boxed{\begin{aligned} &P_1(0, 0) \\ &P_2(-1, -1) \end{aligned}}$$

19 b)

$$\{(x_1, x_2, f(x_1, x_2)) \mid (x_1, x_2) \in \mathbb{R}^2\}$$

$$f(1,1) = 1 + 1 + 3 = 5$$

$$f_{x_1}(1,1) = 3 + 3 = 6$$

$$f_{x_2}(1,1) = 6$$

$$\boxed{x_3 = 5 + 6(x_1 - 1) + 6(x_2 - 1)}$$

c) $P(1, 1, 5)$

$$-\operatorname{grad} f(1, 1, 5) = -(3(1)^2 + 3 \cdot 1 \;,\; 3 \cdot 1^2 + 3)$$

$$= -(6 \;,\; 6)$$

$$\vec{v} = -\begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \\ 1 \end{pmatrix}$$

d)

$$\{(x_1, x_2) \in \mathbb{R}^2 \mid f(x_1, x_2) = 5\}$$

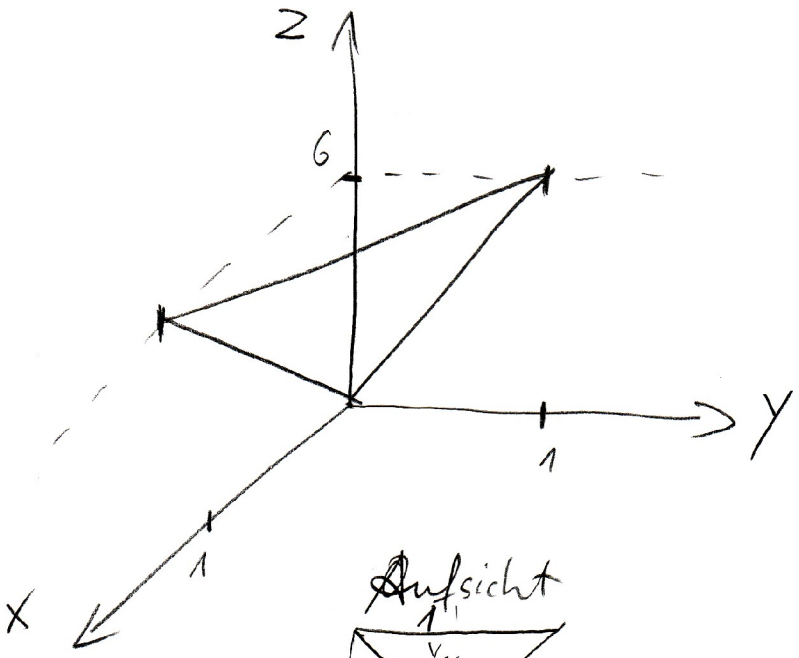
$$x_1^3 + x_2^3 + 3x_1x_2 = 5$$

$P(1, 1)$

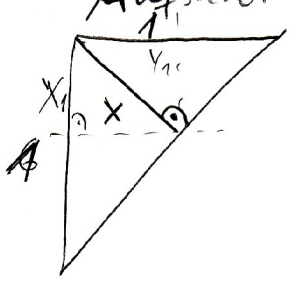
$P_2(1, 1, 5)$

H29 c) $P(1,1,5)$

$\operatorname{grad} f(1,1) = (6, 6)$



Aufsicht

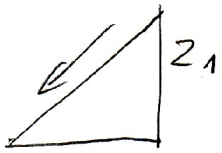


$$x^2 + x^2 = 1$$
$$2x^2 = 1$$
$$x^2 = \frac{1}{2}$$
$$x = \sqrt{\frac{1}{2}} = \underline{\underline{\frac{1}{2}\sqrt{2}}}$$

$x_1 = \frac{1}{2}$

$y_1 = \frac{1}{2}$

$z_1 = 6$

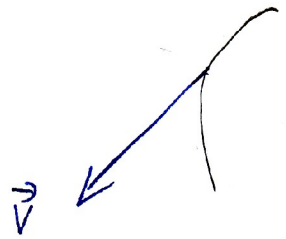


Der Ball rollt in die Richtung

$$\boxed{\vec{v} = \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ -6 \end{pmatrix}}$$

-1

d)



$$\vec{v} = \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ -6 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 1 \\ 12 \end{pmatrix}$$

$$\vec{t} \perp \vec{v}$$

$$\vec{t} \in E$$

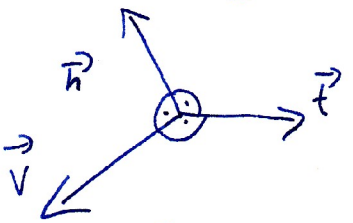
$$\vec{t} = \begin{pmatrix} t_1 \\ t_2 \\ t_3 \end{pmatrix}$$

E: $x_3 = 5 + 6x_1 - 6 + 6x_2 - 6$

$= 5 - 12 + 6x_1 + 6x_2$

$$\boxed{0 = -x_3 + 6x_2 + 6x_1 - 7}$$

$$\left[\vec{x} - \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}\right] \begin{pmatrix} 6 \\ 6 \\ -1 \end{pmatrix} = 0 \qquad \vec{n} = \begin{pmatrix} 6 \\ 6 \\ -1 \end{pmatrix}$$



$$\vec{t} \perp \vec{n}$$

$$\vec{v} \times \vec{n} = \begin{pmatrix} 1 \\ 1 \\ 12 \end{pmatrix} \times \begin{pmatrix} 6 \\ 6 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 - 6 \cdot 12 \\ 12 \cdot 6 + 1 \\ 6 - 6 \end{pmatrix}$$

$$= \begin{pmatrix} -1 - 72 \\ 72 + 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

Damit ist die Tangente:

$$\boxed{T(x) = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix} + x \cdot \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}}$$

,0)

9_A positiv definit

$$
\begin{array}{lccc}
 & 2 & -2 & -6 \\
 & -2 & -3 & 6 \\
 & -6 & 4 & 1 \\
\hline
 & 2 & -2 & -6 \\
\mathrm{I}+\mathrm{II} & 0 & -5 & 0 \\
\mathrm{III}+3\mathrm{I} & 0 & -2 & -17
\end{array}
$$

9_b indefinit

$$
\begin{array}{lccc}
 & 4 & 0 & -2 \\
 & 0 & 2 & 2 \\
 & -2 & 2 & 3 \\
\hline
:2 & 2 & 0 & -1 \\
 & 0 & 2 & 2 \\
\mathrm{III}-\mathrm{II} & -2 & 0 & 1 \\
\hline
 & 2 & 0 & -1 \\
 & & 1 & 1 \\
\mathrm{III}+\mathrm{I} & 0 & 0 & 0
\end{array}
$$

9_c keine Entscheidung möglich

$$f = x^T A x = \frac{1}{3}(x_1, x_2, x_3)\begin{pmatrix} 7 & 2 & 0 \\ 2 & 6 & 2 \\ 0 & 2 & 5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$= \frac{1}{3}\left(7x_1^2 + 4x_1x_2 + 6x_2^2 + 4x_2x_3 + 5x_3^2\right)$$

$f_{x_1} = \frac{1}{3}(14x_1 + 4x_2)$ $\quad f_{x_2} = \frac{1}{3}(4x_1 + 12x_2 + 4x_3)$ $\quad f_{x_3} = \frac{1}{3}(4x_2 + 10x_3)$

$f_{x_1x_1} = \frac{1}{3}(14)$ $\quad f_{x_2x_1} = \frac{1}{3}\cdot 4$ $\quad f_{x_3x_2} = \frac{4}{3}$

$f_{x_1x_2} = \frac{1}{3}\cdot 4$ $\quad f_{x_2x_2} = 4$ $\quad f_{x_3x_3} = \frac{10}{3}$

$f_{x_2x_3} = \frac{4}{3}$

.50

$$H = \begin{pmatrix} \frac{14}{3} & \frac{4}{3} & 0 \\ \frac{4}{3} & 4 & \frac{4}{3} \\ 0 & \frac{4}{3} & \frac{10}{3} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 14 & 4 & 0 \\ 4 & 12 & 4 \\ 0 & 4 & 10 \end{pmatrix}$$

$$= \frac{1}{2} \cdot A$$

$$\cancel{J = \left(\frac{1}{3}(14x_1 + 4x_2) \quad \frac{1}{3}(4x_1 + 12x_2 + 4x_3) \quad \frac{1}{3}(4x_2 + 10x_3) \right)}$$

$$Ax = \begin{pmatrix} 7x_1 + 2x_2 \\ 2x_1 + 6x_2 + 2x_3 \\ 2x_2 + 5x_3 \end{pmatrix}$$

$$J = \begin{pmatrix} 7 & 2 & 0 \\ 2 & 6 & 2 \\ 0 & 2 & 5 \end{pmatrix} \cdot \frac{1}{3} = A$$

31)

a) $$Jf(x,y,z) = \begin{pmatrix} \frac{1}{x} & 1 & 0 \\ 0 & z & y \end{pmatrix}$$

$$Jg(s,t) = \begin{pmatrix} 2s & 2t \\ te^s & e^s \end{pmatrix}$$

b) $$J(g \circ f)(x,y,z)$$

$$= \left(Jg\begin{pmatrix} s \\ t \end{pmatrix}\right)\left(Jf\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right)$$

$$= \begin{pmatrix} 2s & 2t \\ te^s & e^s \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{x} & 1 & 0 \\ 0 & z & y \end{pmatrix} = \begin{pmatrix} \frac{2s}{x} & 2s+2tz & 2ty \\ \frac{te^s}{x} & te^s+e^s z & e^s y \end{pmatrix}$$

$$s = \ln(x) + y$$
$$t = y \cdot z$$

$$= \begin{pmatrix} \frac{2(\ln x+y)}{x} & 2(\ln x+y) + 2 \cdot y \cdot z \cdot z & 2y \cdot z \cdot y \\ \frac{y \cdot z \cdot e^{\ln(x)+y}}{x} & y \cdot z \cdot e^{\ln(x)+y} + e^{(\ln x+y)} \cdot z & e^{\ln x+y} \cdot y \end{pmatrix}$$